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Acceleration Consensus for Networked Motion Control

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1 Introduction

Multiple networked robots can achieve difficult tasks by coordination. However, coordination on a network is also a difficult problem and should be analyzed by mathematical tools. Consensus algorithms are a strong tool to analyze coordination in networked systems. It has been shown in [1] that, distributed robots on a network with different initial positions can agree on a place to meet; or they reach a position consensus. First the topology of the network is represented by graphs as shown in figure 1. A position consensus algorithm can be synthesized using the graph properties of the network. The graphs can be mathematically written using connection matrices. Then we obtain Laplacian matrices to write the dynamics of consensus algorithms. The Laplacian of the graph on figure 1 is:

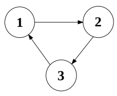


Figure 1: Information Graph

$$L = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \tag{1}$$

The simplest position consensus algorithm can then be written as:

$$\dot{x}_i = \sum_{v_j \in N_i} (x_j - x_i) \tag{2}$$

$$\dot{x} = -Lx \tag{3}$$

If the graph is strongly connected and balanced, it can be shown that, all the positions will converge to the average of the initial positions.

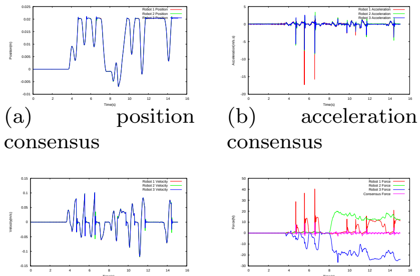
2 Acceleration Consensus

Similarly we introduce acceleration consensus algorithms [2], which can be written as:

$$\ddot{x}_i = \sum_{j \in N_i} K_{p_{ij}} (x_j - x_i) + \sum_{j \in N_i} K_{v_{ij}} (\dot{x}_j - \dot{x}_i) + u_i \tag{4}$$

$$\ddot{x} = -K_p L x - K_v L \dot{x} + u \tag{5}$$

where $K_{p_{ij}}$ and $K_{v_{ij}}$ are positive graph weights and u_i are the local servo input. We have proven that using this algorithm, the robot accelerations will all converge to the average of the inputs and velocities as well as the positions will also be equal. It turns out that this algorithm is very useful for coordination in networked motion control.



(a) position consensus

(b) acceleration consensus

(c) velocity consensus

(d) force consensus

Figure 2: Succesful haptic consensus on a 10Hz switching network with a switching consensus filter

3 Haptic Consensus

Haptics is a network motion control problem which requires coordination. It is actually possible to show that haptic teleoperation in multi robots is also a consensus problem[2]. In haptics when using multiple robots, the goal of the control system is to have:

$$F_1 + F_2 + \ldots + F_N = 0 \tag{6}$$

$$X_1 = X_2 = \ldots = X_N \tag{7}$$

where F_i are the forces felt at each robot and X_i are the positions. This means that the robots should agree on the total force they feel which should be 0, and they should agree on their positions and velocities. Thus we can realize haptic consensus using a special acceleration consensus algorithm with a common input of forces:

$$-\frac{1}{M_n}(G(s)F_{dis1}) + \ddot{x} = -K_p L x - K_v L \dot{x} + \frac{1}{M_n}(F_1 + F_2 + \ldots + F_N) \tag{8}$$

where $G(s)$ is the disturbance sensitivity function. By using disturbance observers that becomes zero and we have a perfect consensus algorithm. This means that the goals are reached. Furthermore the common input can be estimated by using consensus filters, and using these, haptic consensus can be realized on networks with switching topology, which is a very extreme case for testing the stability. Figure 2 shows the experiment results on a 3 robot switching network.

References

[1] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Trans. on Automatic Control*, vol. 49(9), pp. 1520-1533, September 2004.

[2] U. Tumerdem and K. Ohnishi, "Haptic Consensus in Bilateral Teleoperation," *The 4th IEEE International Conference on Mechatronics, ICM '07 K*, TuA1-A5, pp 1-6,May 2007.