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Closed-Loop Identification of Hammerstein-Wiener Systems

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1 Intorduction

Closed-loop identification method based on output inter-sampling scheme is proposed for a class of Hammerstein and Hammertein-Wiener sysytem. One is based on blind system identification and the other is based on the extended Kalman filter algorithm.

2 Problem Statement

For the simplitity, we assume that the SISO model is only addressed. The holding period of T-model(see Figs 1 and 3) is defined as T and the system output is sampled by interval $\Delta = T/p$, where p is a positive integer. Then T-model may be transfered into Δ-model(see Figs 2 and 4). C is a controller. The intermediate output signals$(y_L, y_{L,\Delta}, y_N, y_{N,\Delta})$ are unknown. we will deal with the follow problems:

1)A discrete-time Hammerstein-Wiener system in closed-loop is considered, which consists of two static nonlinear elements N1 and N2 surrounding a linear dynamic model L_T(2.2) where both N1 and N2 are polynomial forms (2.1) and (2.2), The orders of N1 and N2 are ℓ and m, respectively, and assumed to be known. L_T is assumed to be stable.

$$y_N1 = \sum_{i=1}^{m} \Theta_{N1,i} u^i \quad (2.1), \qquad L_T = \frac{B(z^{-1})}{A(z^{-1})} \quad (2.2)$$

$$y_N2 = \sum_{i=1} \Theta_{N2,i} y_L^i \quad (2.3), \qquad L_\Delta = \frac{B_\Delta(q^{-1})}{A_\Delta(q^{-1})} \quad (2.4)$$

where y_L is the output of linear element in Figs 1, 2 and 3.

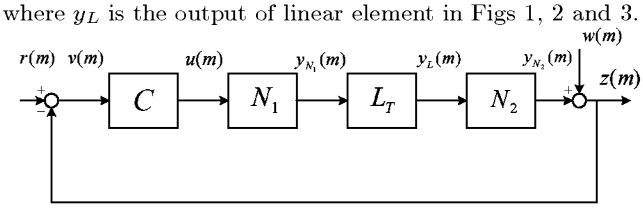

Figure 1: Hammerstein-Wiener system (T-model)

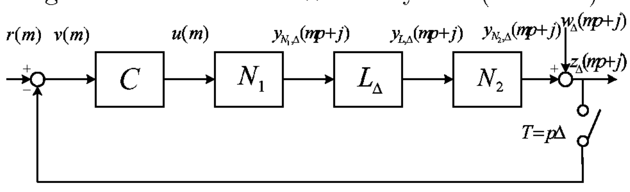

Figure 2: Hammerstein-Wiener system (Δ-model)

2)A discrete-time Hammerstein system in closed-loop is considered, which consists of static nonlinear element N followed linear dynamic element L_T, (see Figs 3 and 4).

The purpose of this thesis is to identify the above non-linear and linear subsystems via the output inter-sampling scheme by only using input and output signal.

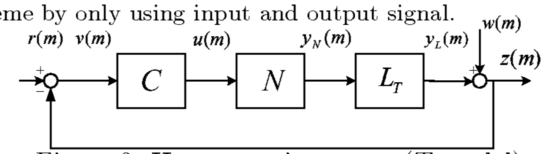

Figure 3: Hammerstein system (T-model)

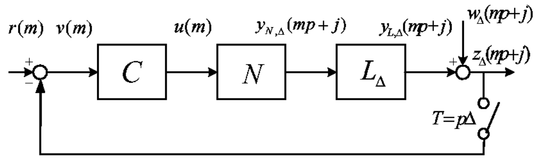

Figure 4: Hammerstein system (Δ-model)

3 The Output Inter-sampling Scheme

In the output inter-sampling scheme, one important condition is that the input signal to a discrete-time system has holding period T, i.e, the holding rate of input is refered to as $1/T$. The output signal is assumed to measure p times during one input holding period. Under the output inter-sampling scheme, the following results can be obtained:

Theorem 3.1: The input signal to a linear discrete-time system is given by a step input. Let $A_\Delta(q^{-1})$ and $B_\Delta(q^{-1})$ be the denominator and numerator of the Δ-model transfer function, and $A(z^{-1})$ and $B_j(z^{-1})$ be the denominator and numerator of the single-input multi-output transfer function (see Figs 5 and 6), then $A(z^{-1})$, $B_j(z^{-1})$ can be given by (3.1).

$$\begin{cases} A(z^{-1}) = det(I - A^p z^{-1}) & (3.1) \\ B_j(z^{-1}) = c^T adj(I - A^p z^{-1})(\sum_{i=0}^{j-1} A^i b + \sum_{i=j}^{p-1} A^i b) \end{cases}$$

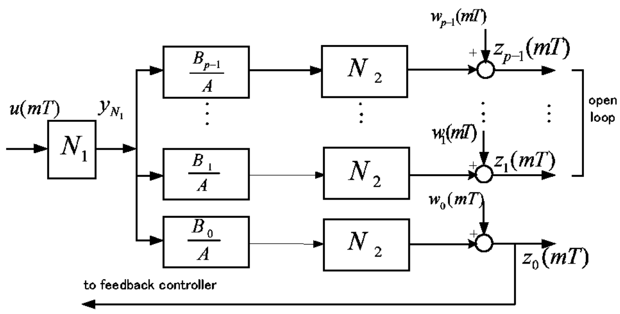

Figure 5: Single-input $p-$output of Hammerstein-Wiener system

Theorem 3.2: The input signal to a linear discrete-time system is given by impulse sequences. Then the system model given by (2.4) can also be described by a single-input $p-$output rational teanfer funvtion model given by (3.2)

$$y_j(m) = \frac{H_j(z^{-1})}{A(z^{-1})} u(m) + e_j(m) \qquad (3.2)$$

Theorem 3.3: The input and output relation of a FIR system is given in (3.3), then the system model given by (3.3) can also described by a single-input p-output FIR model (3.4).

$$y_\Delta(k) = L_\Delta(q^{-1}) u_\Delta(k) + e_\Delta(k)$$

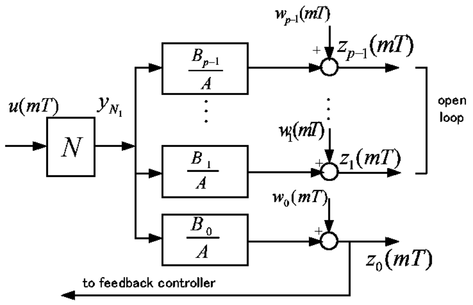

Figure 6: Single-input $p-$output of Hammerstein system

$$\approx (h_1 q^{-1}_{j} + \cdots + h_n q^{-n}_{p=n}) u_\Delta(k) + e_\Delta(k) \qquad (3.3)$$

$$L_{T,j}(z^{-1}) = \sum_{i=1} h_i + z^{-1} \sum_{i=j+1} h_i \qquad (3.4)$$

where n is assumed to be known.

Theorem 3.4: Let the input-output relation to an FIR system is given in (3.4), then the parameters of (3.4) can be estimated if and only if $p \geq n$ holds.

4 EKF Identification

Recall the multi-output subsystems in the output inter-sampling scheme. Here, it will be assumed that a model of the same order and structure as that of the true system is used for identification algorithm based on EKF, which is summeried as:

Step-1: Estimation of $N1$, $N2$ and $L_\Delta(q^{-1})$:

Based on Theorems 3.3 and 3.4, and by using EKF algorithm, the parameters of $N1$, $N2$ and $L_\Delta(q^{-1})$ are estimated.

Step-2: Estimation of $L_T(z^{-1})$:

By making use of the approximation method, $L_T(z^{-1})$ can be obtained from $L_\Delta(q^{-1})$.

5 Numerical Examples

The output sampling interval is given by Δ=0.2 and the system input u holding a duration time is set by T=2.4, then the input-holding factor is p=12. Assume that the parameters of unknown true system with the sampling interval Δ are given by

$$\Theta^T_{N1} = [0.5, 0.3], \qquad \Theta^T_{N2} = [0.4, 0.5]$$

$$\Theta^T_{l,\Delta} = [0.7, 1.1.49, 0.7849, 0.0220, -0.3890, -0.4022, -0.1688, 0.0725, 0.1738, 0.1303, 0.0260, -0.0522, \cdots]$$

where $\Theta_{N1,1}$=$\Theta_{N2,1}$=1 is known as *a priori*. The noise is white noise and the total observation time is $500T$. The simulations were performed 20 times. Then, the estimated parameters are given by

$$\hat{\Theta}^T_{N1} = [0.5067 \pm 0.0094, 0.3002 \pm 0.0001]$$

$$\hat{\Theta}^T_{N2} = [0.4173 \pm 0.0302, 0.5018 \pm 0.0017]$$

$$\begin{aligned}\hat{\Theta}^T_{l,\Delta} = [&0.6900 \pm 0.0225, 1.055 \pm 0.0228 \\ &0.7325 \pm 0.0224, 0.0216 \pm 0.0015 \\ &-0.3896 \pm 0.0085, -0.4024 \pm 0.0020 \\ &-0.1687 \pm 0.0035, 0.0727 \pm 0.0010 \\ &0.1740 \pm 0.0050, 0.1299 \pm 0.0045 \\ &0.0263 \pm 0.0008, -0.0699 \pm 0.0011]\end{aligned}$$

6 Algorithm Based On Blind Identification

In order to identify linear and nonlinear parts independently and sufficiently, a new joint blind and closed-loop identification method is investigated. An iterative identification method via output inter-sampling scheme is proposed. Recalling the multi-output subsystems in the output inter-sampling scheme. The new algorithm is given as follows:

Step-1: Based on Theorem 3.2 and by using the subspace method for blind identification, $L_\Delta(q^{-1})$ can be estimated. Set this solution as an initial value.

Step-2: Based on Theorem 3.1, take transform of $L_\Delta(q^{-1})$($SISO\Delta$-model) into a single-input p-output subsystems.

Step-3: By using the above multi-output subsystems, restore the intermediate output $y_{N,\Delta}$.

Step-4: By using the above solution and input signal, estimate nonlinear element.

Step-5: By using the estimated nonlinear element, renew the intermediate output $y_{N,\Delta}$.

Step-6: By using the renewed intermediate output $y_{N,\Delta}$ and system output, estimate linear part $L_\Delta(q^{-1})$.

Step-7: Return to Step-2 if not convergence.

7 Numerical Examples

Assume that the linear parameters of unknowm true system with p=5 are given by

$$\Theta^T_{\Delta,a} = [1.0000, -1.1771, 0.5824, -0.1466]$$
$$\Theta^T_{\Delta,b} = [6.2157, 2.1800, 0.8990]$$

Assume that nonlinearity is given by

$$x = \begin{cases} 4.2 & u \geq 6.3 \\ u/(1+0.0315u^2)^{1/2} & -7.71 < u < 6.3 \\ -4.55 & u \leq -7.71 \end{cases}$$

Then, the estimated linear parameters are given by

$$\begin{aligned}\hat{\Theta}^T_{\Delta,a} = &[1.0000, -1.1775 \pm 0.0239, \\ &0.5829 \pm 0.0217, -0.1470 \pm 0.0226] \\ \hat{\Theta}^T_{\Delta,b} = &[6.2088 \pm 0.0667, 2.1738 \pm 0.0860, \\ &0.9014 \pm 0.0555]\end{aligned}$$

The estimated nonlinearity is shown in Fig.7.

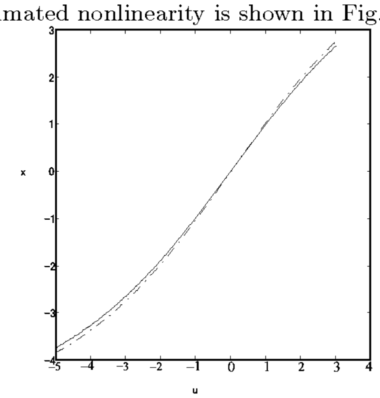

Figure 7: Estimated nonlinearity, solid line: true; dashed line: estimate

8 Conclusion

In this thesis,the two identification methods based on output inter-sampling scheme is presented for Hammerstein and Hammerstein-Wiener systems inaclosed-loop control system.